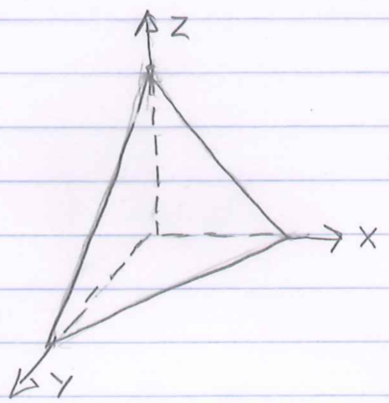
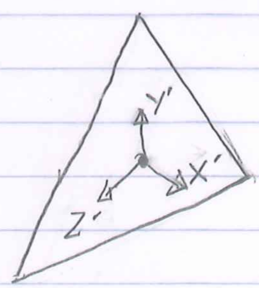
26. Stress transformation: 3-d stress states

The previous relations for $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$ in terms of σ_x, σ_y, σ_z and θ are valid even if σ_y, τ_{yz}, τ_{xz} are present. Can you show this?

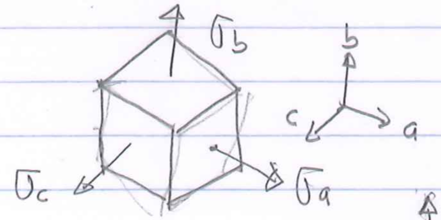
There is a general 3-d transformation for the x', y', z' stresses in terms of the x, y, z stresses and the direction cosines of the x', y', z' axes. This comes from considering the equilibrium of a differential tetrahedron.



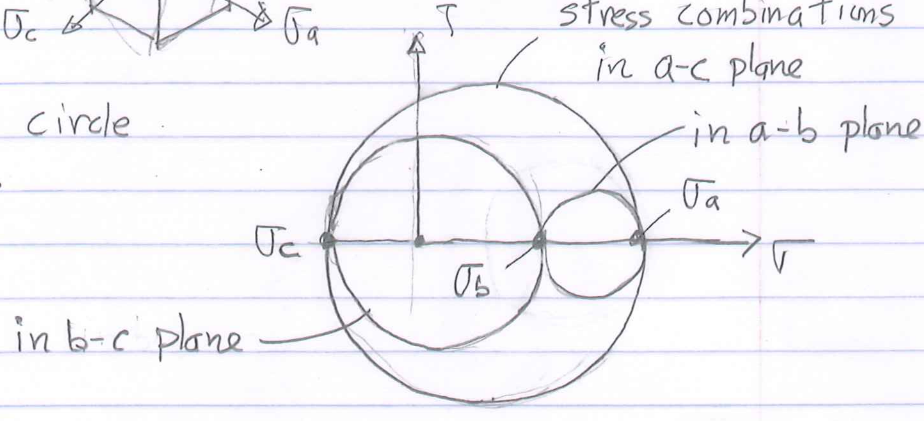


x' is normal to the inclined face

One finds that there is always a set of axes x', y', z' for which all shear stresses vanish, leaving only normal stresses. Denote these axes by a, b, c and the normal stresses by σ_a, σ_b and σ_c, ordered $\sigma_a \geq \sigma_b \geq \sigma_c$. These are principle stresses.



Consider three Mohr's circle plots shown at right.



It can be shown that

Maximum σ on any face is σ_a on face a

Minimum σ on any face is σ_c on face c

Maximum τ on any face has magnitude $\frac{1}{2}(\sigma_a - \sigma_c)$ and occurs on faces rotated $45°$ about axis b from faces a and c.

Example: $\sigma_x = 50$ MPa, $\sigma_y = -10$ MPa, $\sigma_z = -50$ MPa

$\tau_{xy} = 40$ MPa, $\tau_{yz} = 0$, $\tau_{xz} = 0$

The principal stresses are
$$\sigma_a = 70 \text{ MPa}$$
$$\sigma_b = -30 \text{ MPa}$$
$$\sigma_c = -50 \text{ MPa}$$

The maximum, minimum σ's on any face are 70 MPa, −50 MPa.

The maximum τ on any face has magnitude

$$\frac{1}{2}(70 \text{ MPa} + 50 \text{ MPa}) = 60 \text{ MPa}.$$